THE LAWS OF LOGS

❑ INTRODUCTION

Since a log is really just an exponent, and since we have laws for exponents, it makes sense that we need to learn some laws for logs, too. These laws of logs will be used to solve new kinds of equations. This chapter also shows us another application of logs, the Richter scale, for measuring the magnitude of an earthquake.

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Richter 7.8

❑ THE TWO CANCELLATION RULES

Carefully study the next five examples:

$$\log_9 9^2 = \log_9 81 = 2$$

$$\log_{10} 10^5 = \log_{10} 100{,}000 = 5$$

$$\log_5 5^3 = \log_5 125 = 3$$

$$\log_2 2^1 = \log_2 2 = 1$$

$$\log_e e^0 = \log_e 1 = 0$$

Something's going on here. In each problem, the final answer matches the exponent at the front of the problem. For example,

$$\log_9 9^{\boxed{2}} = \boxed{2}$$

Also notice that the base of the log matches the base of the expression that we're taking the log of. For instance,

$$\log_{\boxed{10}} \boxed{10}^{5} = 5$$

Using these two insights, we might now see that $\log_{12} 12^7 = 7$. We can now generalize this whole discussion into the first of two cancellation rules:

$$\boxed{\log_b b^x = x}$$

We call it a canceling rule because the log function (done second) cancels out the exponential function (done first), leaving just the *x*.

To motivate the second cancellation rule, consider the following four calculations:

$$10^{\log_{10} 1000} = 10^3 = 1000$$

$$2^{\log_2 32} = 2^5 = 32$$

$$5^{\log_5 125} = 5^3 = 125$$

$$7^{\log_7 1} = 7^0 = 1$$

For each example, notice that the base of the entire question matches the base of the log — for example, $\boxed{7}^{\log_{\boxed{7}} 1}$. We also see that in every case, the final answer matches the number we're taking the log of — for instance, $2^{\log_2 \boxed{32}} = 2^5 = \boxed{32}$. In a nutshell,

$$\boxed{b^{\log_b x} = x}$$

This is a canceling rule because the exponential function (done second) cancels out the log function (done first), leaving just the *x*.

EXAMPLE 1: **Use the canceling rules to simplify each expression:**

A. $\log_3 3^n = n$ B. $\log 10^{x+y} = x+y$

C. $7^{\log_7(ab)} = ab$ D. $e^{\ln(e+4)} = e+4$

E. $\log_4 5^n$ cannot be simplified by a canceling rule, since the base of the exponential (the 5) does not match the base of the log (the 4).

F. $8^{\log 7}$ also cannot be simplified by a canceling rule, since the base of the exponential (the 8) does not match the base of the log (the 10).

Homework

1. In the second canceling rule, explain why we must restrict *x* to the positive real numbers.

2. Simplify each expression:

a. $\log 10$	b. $\ln e$	c. $\log 10^{u+v}$	d. $\ln e^{xyz}$
e. $\log_9 9^{500}$	f. $10^{\log(xy)}$	g. $e^{\ln(\log 7)}$	h. $2^{\log_2(\ln e)}$
i. $3^{\log_3(a-b)}$	j. $\log_2 2^R$	k. $\log e^x$	l. $e^{\log Q}$

3. A common student error is to figure that $\log_b(x+y) = \log_b x + \log_b y$. We need to dispel this myth right now. Let b = 2, so that we're working with base 2. Then let both

x and y equal 8. Show that the left side of the formula results in 4, whereas the right side comes out 6.

4. Prove that the conjecture log (xy) = (log x)(log y) is false. Hint: Let x = 100 and y = 1000 and work out each side.

❑ THE SUM OF LOGS

We know how to calculate certain logs in our heads; for example, $\log_5 25 = 2$, because $5^2 = 25$. Sometimes in a problem we have the sum or difference of two logs, and our goal is to convert that sum or difference into a single log. For instance, here's how we can

Convert $\log 100 + \log 1000$ *into a single common log.*

Start with the sum of the logs:	$\log 100 + \log 1000$
Calculate each log separately:	$2 \quad + \quad 3$
And add the numbers:	5
Now we have to write 5 as the common log of something.	
Since 10^5 = 100,000, we can finish by writing	$\log 100{,}000$

In short, $\log 100 + \log 1000 = \log 100{,}000$

For a second example, let's

Find the sum of $\ln e^5$ *and* $\ln e^3$.

Start with the sum of the logs:	$\ln e^5 + \ln e^3$
Calculate each log separately:	$5 \quad + \quad 3$
And add the numbers:	8
Now we have to write 8 as the natural log of something.	$\ln e^8$

That is, $\ln e^5 + \ln e^3 = \ln e^8$

Homework

5. Express each sum of logs as a single log:

a. $\log 10 + \log 1000$

b. $\log 10{,}000 + \log 100$

c. $\log 10 + \log 1$

d. $\log 1000 + \log 1000$

e. $\log_2 8 + \log_2 64$

f. $\log_3 9 + \log_3 3$

g. $\log_4 64 + \log_4 4 + \log_4 1$

h. $\log 10 + \log 100 + \log 1000$

i. $\ln e^5 + \ln e^7$

j. $\ln e^{10} + \ln e^{10}$

❑ *The Difference of Logs*

To find the difference of logs, we use the same logic as above:

Express $\log_2 32 - \log_2 8$ *as a* *<u>single</u>* *log.*

$\log_2 32 - \log_2 8$

$= \quad 5 - 3$ (since $2^5 = 32$ and $2^3 = 8$)

$= \quad 2$ (arithmetic)

$= \quad \log_2 4$ (since $2^2 = 4$)

In short, $\log_2 32 - \log_2 8 \;=\; \log_2 4$

For a second difference of logs problem,

Express $\ln e^9 - \ln e^6$ *as a single log.*

$\ln e^9 - \ln e^6$

$= \quad 9 - 6$ (take the individual logs)

$= \quad 3$ (arithmetic)

$= \quad \ln e^3$ (write 3 as an ln)

So, $\ln e^9 - \ln e^6 \;=\; \ln e^3$

Homework

6. Express each difference of logs as a single log:

a. $\log 100{,}000 - \log 1000$

b. $\log 1000 - \log 10$

c. $\log_5 125 - \log_5 25$

d. $\ln e^{10} - \ln e^{7}$

e. $\log_6 36 - \log_6 6$

f. $\ln e^{20} - \ln e^{5}$

g. $\log 1000 - \log 100$

h. $\log_8 512 - \log_8 64$

i. $\log_2 32 - \log_2 8$

j. $\log_{12} 144 - \log_{12} 1$

❑ *Synopsis of the Two Preceding Sections*

Let's look at two of the results from the preceding sections:

1) $\log 100 + \log 1000 = \log 100{,}000$

This shows that the sum of two logs can be combined into a single log — as long as the numbers are multiplied.

2) $\log_2 32 - \log_2 8 = \log_2 4$

This shows that the difference of two logs can be combined into a single log — as long as the numbers are divided.

These two results, that the sum of logs is the log of the product, and that the difference of logs is the log of the quotient, can be written like this:

$$\log_b x + \log_b y = \log_b (xy)$$

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$

Traditionally, these two laws of logs are written the other way around; indeed, that's the way we need one of them in the next section.

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

The log of a product is the sum of the logs.

The log of a quotient is the difference of the logs.

EXAMPLE 4: **Use the two Laws of Logs to expand each expression:**

A. $\log(ABC) = \log A + \log B + \log C$

B. $\ln\left(\frac{xy}{z}\right) = \ln(xy) - \ln z = \ln x + \ln y - \ln z$

C. $\log_2\left(\frac{a}{bc}\right) = \log_2 a - \log_2(bc) = \log_2 a - (\log_2 b + \log_2 c)$

$= \log_2 a - \log_2 b - \log_2 c$

❑ *THE LOG OF A POWER*

Another type of log expression we'll come across is the log of a power, e.g., $\log(7^3)$. We can break it down and use the notion of exponent and the "log of a product" law above to do the problem. Let's work $\log 7^3$:

	$\log 7^3$	
$=$	$\log(7 \cdot 7 \cdot 7)$	(definition of exponent)
$=$	$\log 7 + \log 7 + \log 7$	(use the "log of a product" law)
$=$	$3\log 7$	(combine like terms)

This may not seem very profound, but this maneuver will be necessary to solve exponential equations later in the course. Let's do one more example, and we'll even put a variable into the expression.

$$\ln x^4 = \ln(x \cdot x \cdot x \cdot x) = \ln x + \ln x + \ln x + \ln x = 4\ln x$$

This "log of a power" law can be summarized as

$$\log_b x^n = n\log_b x$$

To calculate the log of a power, bring down the power as a coefficient.

❑ SUMMARY OF LOGS

We can summarize our knowledge of **logs** in the following six statements — one definition, two cancellation rules, and three laws:

Statement	Example
$y = \log_b x$ means $b^y = x$	$2 = \log 100$ means $10^2 = 100$
$\log_b b^x = x$	$\ln e^T = T$
$b^{\log_b x} = x$	$2^{\log_2 99} = 99$
$\log_b(xy) = \log_b x + \log_b y$	$\log(100x) = 2 + \log x$
$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\ln \frac{x}{e} = \ln x - 1$
$\log_b x^n = n\log_b x$	$\log_5 \sqrt[7]{Q} = \frac{1}{7}\log_5 Q$

EXAMPLE 5: **Use one of the Laws of Logs to expand each expression:**

A. $\log(7x) = \log 7 + \log x$

B. $\ln(xyz) = \ln x + \ln y + \ln z$

C. $\ln(ex) = \ln e + \ln x = 1 + \ln x$

D. $\log_2 \frac{8}{3} = \log_2 8 - \log_2 3 = 3 - \log_2 3$

E. $\log \frac{A}{B} = \log A - \log B$

F. $\log_{12}[(a+b)(a-b)] = \log_{12}(a+b) + \log_{12}(a-b)$

G. $\log(x^3) = 3\log x$

H. $\log_3 \sqrt{x} = \log_3 x^{1/2} = \frac{1}{2}\log_3 x$

I. $\ln \sqrt[3]{w^2} = \ln w^{2/3} = \frac{2}{3}\ln w$

J. $(\ln x)^2$ cannot be simplified because the power is on the ln x, not the x. Note that $\ln x^2 = 2\ln x$. See the difference?

EXAMPLE 6: **Use the Laws of Logs to expand each expression:**

A. $\ln(ax^3) = \ln a + \ln x^3 = \ln a + 3\ln x$

B. $\log \frac{xy}{z} = \log(xy) - \log z = \log x + \log y - \log z$

C. $\ln \frac{a}{bc} = \ln a - \ln(bc) = \ln a - [\ln b + \ln c] = \ln a - \ln b - \ln c$

D. $\log_2 \frac{cd^3}{e^4} = \log_2 cd^3 - \log_2 e^4 = \log_2 c + 3\log_2 d - 4\log_2 e$

E. $$\ln\left(\frac{\sqrt{x}\ \sqrt[3]{y}}{e^z\ \sqrt[5]{w^3}}\right) = \ln\sqrt{x}\ \sqrt[3]{y} - \ln e^z\ \sqrt[5]{w^3}$$

$$= \ln\sqrt{x} + \ln\sqrt[3]{y} - \left[\ln e^z + \ln\sqrt[5]{w^3}\right]$$

$$= \tfrac{1}{2}\ln x + \tfrac{1}{3}\ln y - z - \tfrac{3}{5}\ln w$$

<u>EXAMPLE 7:</u> **Use the Laws of Logs to condense each expression into a single log with coefficient 1:**

A. $\log x + \log y = \log(xy)$

B. $\ln a - \ln b = \ln\dfrac{a}{b}$

C. $3\log_2 T = \log_2 T^3$

D. $\tfrac{3}{7}\log n = \log n^{3/7} = \log\sqrt[7]{n^3}$

E. $\tfrac{1}{2}\ln x + 5\ln y = \ln\sqrt{x} + \ln y^5 = \ln\left(\sqrt{x}\, y^5\right)$

F. $2\log x + \tfrac{1}{2}\log y + \tfrac{2}{5}\log z = \log x^2 + \log\sqrt{y} + \log\sqrt[5]{z^2}$

$$= \log\left(x^2\sqrt{y}\ \sqrt[5]{z^2}\right)$$

G. $\ln x + \ln y - \ln z = \ln\dfrac{xy}{z}$

H. $2\log x - \tfrac{1}{3}\log y = \log x^2 - \log y^{1/3} = \log\dfrac{x^2}{\sqrt[3]{y}}$

I. $\ln a - \ln b - \ln c = \ln\dfrac{a}{b} - \ln c = \ln\dfrac{\frac{a}{b}}{c} = \ln\dfrac{a}{bc}$

Alternate approach:

$$\ln a - \ln b - \ln c = \ln a - (\ln b + \ln c) = \ln a - \ln(bc) = \ln\frac{a}{bc}$$

J. $\frac{2}{3}\log(a+b)-\frac{1}{2}\log(a-b)-7\log(ab)$

$$= \log\sqrt[3]{(a+b)^2}-\log\sqrt{a-b}-\log(ab)^7$$

$$= \log\frac{\sqrt[3]{(a+b)^2}}{\sqrt{a-b}}-\log a^7b^7$$

$$= \log\frac{\frac{\sqrt[3]{(a+b)^2}}{\sqrt{a-b}}}{a^7b^7}$$

$$= \log\frac{\sqrt[3]{(a+b)^2}}{a^7b^7\sqrt{a-b}}$$

Homework

7. Expand each expression:

a. $\ln(ab)$ b. $\log(wyz)$ c. $\log_2(abcd)$

d. $\log\frac{h}{k}$ e. $\log\frac{a+b}{c}$ f. $\log_8\frac{x-y}{u+v}$

g. $\ln x^3$ h. $\log 7^n$ i. $\log_5 5^7$

8. Expand each expression:

a. $\log_7\sqrt{x}$ b. $\log\sqrt[3]{x}$ c. $\ln\sqrt[5]{Q^2}$

d. $\log\frac{1}{x^3}$ e. $\ln\frac{1}{u^{3/2}}$ f. $\log\frac{1}{T^{-4/5}}$

9. Expand each expression:

a. $\ln(a^2b)$ b. $\log(xy^3)$ c. $\ln(a^2b^3c^4)$

d. $\log_6 \dfrac{x^2}{y^3}$ e. $\ln \dfrac{ab}{c^9}$ f. $\log \dfrac{x^2y^5}{z^{10}}$

10. Expand each expression:

a. $\log_3 \dfrac{a}{bc}$ b. $\ln \dfrac{x^2y}{wz^3}$ c. $\log \dfrac{\sqrt{x}}{yz}$

d. $\log(a^2\, b\sqrt[7]{c})$ e. $\ln\left[\dfrac{wx}{yz}\right]^5$ f. $\log_5 \dfrac{a^2\sqrt[3]{y}}{wx}$

11. Condense each expression:

a. $\ln x - \ln 7$ b. $\log y + \log 12$

c. $\frac{1}{3}\ln 4 + \ln 2$ d. $\frac{2}{5}\log t - \frac{1}{5}\log t$

e. $\ln(x^2) + \ln x + \ln 7$ f. $\log x - \log y + \log z$

g. $\ln a - \ln b - 3\ln c$ h. $2\log a - \frac{1}{2}\log b - \frac{2}{3}\log c$

12. Evaluate each expression <u>without a calculator</u>:

a. $\log 10^{100}$ b. $e^{\ln 7}$ c. $\log 5 + \log 2$

d. $\ln \dfrac{e^{10}}{e^2}$ e. $\log_{24} 8 + \log_{24} 3$ f. $\ln x + \ln \dfrac{1}{x}$

g. $\log 50 + \log 20$ h. $\log_3 \frac{1}{2} + \log_3 162$ i. $\log_5 50 - \log_5 2$

13. True or False:

a. $\log 10^9 = 9$ b. $7^{\log_7 R} = 7^R$

c. $\ln e^{abc} = \ln(abc)$ d. $\ln(xy) = \ln x + \ln y$

e. $\log a^b = (\log a)^b$ f. $\ln \dfrac{a}{b} = \dfrac{\ln a}{\ln b}$

g. $\ln(x+y) = \ln x + \ln y$ h. $\log(a-b) = \log a - \log b$

i. $\ln \dfrac{1}{x} = -\ln x$ j. $\log_{12}(x^a b) = (a\log_{12} x)(\log_{12} b)$

k. $\ln(xy^z) = z\ln(xy)$

l. $\log_3 x - \log_3 y = \log_3 \frac{x}{y}$

m. $\log a + \log b = \log(ab)$

n. $\ln e^t = t$

o. $10^{\log 9} = 9$

p. $\ln \frac{1}{x} = \frac{1}{\ln x}$

q. $e^{\ln(e-3)} = e - 3$

r. $\ln \frac{x}{y} = \ln x - \ln y$

s. $\log(ab^n) = \log a + n\log b$

t. $\ln \frac{a^2}{b^5} = \frac{2\ln a}{5\ln b}$

14.** Prove that $\ln(x^8 + 8x^6 + 24x^4 + 32x^2 + 16) = 4\ln(x^2 + 2)$

❑ *THE RICHTER SCALE FOR EARTHQUAKES*

In 1935, Charles Richter devised a scale for earthquakes called the Richter scale (what a coincidence!). If E represents the energy (in joules) of the earthquake, then the Richter magnitude M is given by

$$\boxed{M = \frac{2}{3}\log\left(\frac{E}{2.5\times 10^4}\right)}$$

<u>EXAMPLE 8:</u> **The energy release of the 1906 San Francisco earthquake was 5.96 × 10^16 joules. Find the Richter magnitude of the quake.**

<u>Solution:</u> According to Richter's formula,

$$\begin{aligned} M &= \frac{2}{3}\log\left(\frac{5.96\times 10^{16}}{2.5\times 10^4}\right) \\ &= \frac{2}{3}\log\left(2.384\times 10^{12}\right) \\ &= \frac{2}{3}(12.3773) \\ &= \boxed{8.25} \end{aligned}$$

Homework

15. An earthquake releases 1.75×10^{11} joules of energy. What is the Richter magnitude of the quake?

16. A more serious quake releases 100 times as much energy as the one in the previous problem. Find the Richter magnitude.

❑ *PROOFS OF THE THREE LAWS OF LOGS*

The laws of logs we've developed and used in this chapter were arrived at using a few examples and noting a consistent pattern. We know that all the examples in the world never constitute a genuine proof, so now it's time to set the record straight and prove the laws the right way. There's no homework for this section, so ask your instructor what you're responsible for on the test.

THEOREM: **The First Law of Logs**

$$\log_b(xy) = \log_b x + \log_b y$$

PROOF: Let $w = \log_b x$ and $z = \log_b y$. Then

$x = b^w$ and $y = b^z$ (from the definition of log).

To prove the First Law of Logs, we'll begin with the left-hand side of the equation, substitute the results just obtained, and work our way to the right-hand side of the equation.

	$\log_b(xy)$	(the left side of the formula)
$=$	$\log_b(b^w \cdot b^z)$	(substituting from the results above)
$=$	$\log_b(b^{w+z})$	(the first law of exponents)

$=$	$w + z$	(the first log cancellation rule)	
$=$	$\log_b x + \log_b y$	(substituting back)	**Q.E.D.**

"The log of a <u>product</u> is the <u>sum</u> of the logs."

<u>THEOREM:</u> **The Second Law of Logs**

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

<u>PROOF:</u> Let $w = \log_b x$ and $z = \log_b y$. Then $x = b^w$ and $y = b^z$. We proceed like the previous proof:

	$\log_b\left(\frac{x}{y}\right)$	(the left side of the formula)	
$=$	$\log_b\left(\frac{b^w}{b^z}\right)$	(substituting from the results above)	
$=$	$\log_b(b^{w-z})$	(a law of exponents)	
$=$	$w - z$	(the first log canceling rule)	
$=$	$\log_b x - \log_b y$	(substituting back)	**Q.E.D.**

"The log of a <u>quotient</u> is the <u>difference</u> of the logs."

The Third Law of Logs uses yet another law of exponents. This law is used whenever there's an exponent on the quantity we're taking the log of.

<u>THEOREM:</u> **The Third Law of Logs**

$$\log_b x^n = n\log_b x$$

PROOF: As in the two previous proofs, let $w = \log_b x$, which implies that $x = b^w$. Thus,

	$\log_b x^n$	(the left side of the formula)
$=$	$\log_b (b^w)^n$	(substituting the expression for x)
$=$	$\log_b (b^{wn})$	(one of the laws of exponents)
$=$	wn	(the first log cancelling rule)
$=$	nw	(commutative property for multiplication)
$=$	$n \log_b x$	(substituting back) **Q.E.D.**

"To calculate the log of a power,
bring down the power as a coefficient."

Practice Problems

17. $\ln e + \log 1 - \log_3 3 - \log_7 49 \ =$

18. a. $\log_{12} 12^N \ =$ b. $9^{\log_9 56} \ =$
c. $\ln e^{a-b} \ =$ d. $\log 10^{25} \ =$
e. $\log_3 5^y \ =$ f. $5^{\log_5 (-5)} \ =$

19. $\log_b b^x \quad b^{\log_b x} \ =$

20. $\ln(\log 10) \ =$

21. Expand: $\log\left(\dfrac{ab}{10}\right)$

$y = \log_b x$ means $b^y = x$
$\log_b b^x = x$
$b^{\log_b x} = x$
$\log_b(xy) = \log_b x + \log_b y$
$\log_b \dfrac{x}{y} = \log_b x - \log_b y$
$\log_b x^n = n \log_b x$

22. Expand: $\ln\left(\frac{x^3}{y\sqrt{z}}\right)$

23. Condense: $\ln x + \ln y - \ln z$

24. Condense: $3\log x + \frac{2}{3}\log y - \frac{1}{2}\log z$

25. True/False:

a. $\ln(a-b) = \ln a - \ln b$

b. $\log e^t = t$

c. $\ln\left(\frac{2}{x}\right) = \frac{\ln 2}{\ln x}$

d. $\ln(ab^{10}) = 10\ln(ab)$

26. The energy release of an earthquake is 1.23×10^{17} joules. Use the formula $M = \frac{2}{3}\log\left(\frac{E}{2.5\times10^4}\right)$ to find the Richter magnitude of the quake.

27. True/False:

a. $\log 10^{a+b} = a+b$

b. $\log e^{u-w} = u-w$

c. $6^{\log_6 z} = z$

d. $e^{\log Q} = Q$

e. $\ln(ab) = \ln a + \ln b$

f. $\ln\frac{x}{y} = \frac{\ln x}{\ln y}$

g. $\log ab^y = y\log ab$

h. $\log ab^y = \log a + y\log b$

i. $\ln p - \ln q = \ln\frac{p}{q}$

j. $\log_3 32 - \log_3 8 = \log_3 4$

k. $\ln(x+y+z) = \ln x + \ln y + \ln z$

l. $\log 7^7 = 7\log 7$

m. $(\ln x)^3 = 3\ln x$

n. $\frac{2}{3}\log x + 4\log y = \log\left(\sqrt[3]{x^2}\, y^4\right)$

o. Use your calculator to determine if $\log_3 20 = \frac{\ln 20}{\ln 3}$.

p. If the energy release of an earthquake is 2.807×10^{15} joules, then the Richter number of the earthquake is about 7.37.

Solutions

1. Because the first operation in $b^{\log_b x}$ is $\log_b x$, whose domain is $(0, \infty)$.

2. a. 1 b. 1 c. $u + v$ d. xyz e. 500 f. xy g. log 7 h. 1 i. $a - b$ j. R k. As is l. As is

3. $\log_2(8+8) = \log_2 16 = 4$, whereas
$\log_2 8 + \log_2 8 = 3 + 3 = 6$

4. $\log(xy) = \log(100 \cdot 1{,}000) = \log 100{,}000 = \mathbf{5}$, but
$(\log x)(\log y) = (\log 100)(\log 1{,}000) = 2 \cdot 3 = \mathbf{6}$.

5. a. log 10,000 b. log 1,000,000 c. log 10 d. 1,000,000 e. $\log_2 512$ f. $\log_3 27$ g. $\log_4 256$ h. log 1,000,000 i. $\ln e^{12}$ j. $\ln e^{20}$

6. a. log 100 b. log 100 c. $\log_5 5$

d. $\ln e^3$ e. $\log_6 6$ f. $\ln e^{15}$
g. $\log 10$ h. $\log_8 8$ i. $\log_2 4$
j. $\log_{12} 144$

7. a. $\ln a + \ln b$ b. $\log w + \log y + \log z$
c. $\log_2 a + \log_2 b + \log_2 c + \log_2 d$ d. $\log h - \log k$
e. $\log(a+b) - \log c$ f. $\log_8(x-y) - \log_8(u+v)$
g. $3\ln x$ h. $n\log 7$ i. 7

8. a. $\frac{1}{2}\log_7 x$ b. $\frac{1}{3}\log x$ c. $\frac{2}{5}\ln Q$
d. $-3\log x$ e. $-\frac{3}{2}\ln u$ f. $\frac{4}{5}\log T$

9. a. $2\ln a + \ln b$ b. $\log x + 3\log y$
c. $2\ln a + 3\ln b + 4\ln c$ d. $2\log_6 x - 3\log_6 y$
e. $\ln a + \ln b - 9\ln c$ f. $2\log x + 5\log y - 10\log z$

10. a. $\log_3 a - \log_3 b - \log_3 c$ b. $2\ln x + \ln y - \ln w - 3\ln z$
c. $\frac{1}{2}\log x - \log y - \log z$ d. $2\log a + \log b + \frac{1}{7}\log c$
e. $5\ln w + 5\ln x - 5\ln y - 5\ln z$ f. $2\log_5 a + \frac{1}{3}\log_5 y - \log_5 w - \log_5 x$

11. a. $\ln\frac{x}{7}$ b. $\log(12y)$ c. $\ln(2\sqrt[3]{4})$ d. $\log\sqrt[5]{t}$
e. $\ln(7x^3)$ f. $\log\frac{xz}{y}$ g. $\ln\frac{a}{bc^3}$ h. $\log\frac{a^2}{\sqrt{b}\sqrt[3]{c^2}}$

12. a. 100 b. 7 c. 1 d. 8 e. 1
f. 0 g. 3 h. 4 i. 2

13. a. T b. F c. F d. T e. F f. F g. F h. F i. T
j. F k. F l. T m. T n. T o. T p. F q. F (tricky!)
r. T s. T t. F

14. Hint: Factor on the left side, or use the Third Law of Logs on the right side.

15. $M = 4.6$

16. $M = 5.9$. Notice that this magnitude is only 1.3 Richter points higher than the previous answer, yet the earthquake was 100 times more powerful. Even a small difference in the Richter magnitude represents a huge difference in the actual power of the earthquake.

17. -2

18. a. N b. 56 c. $a - b$ d. 25 e. As is f. Undefined

19. x^2 **20**. 0 **21**. $\log a + \log b - 1$

22. $3\ln x - \ln y - \frac{1}{2}\ln z$ **23**. $\ln\left(\frac{xy}{z}\right)$

24. $\log\left[\frac{x^3\sqrt[3]{y^2}}{\sqrt{z}}\right]$ **25**. They're all false. **26**. 8.46

27. a. T b. F c. T d. F e. T f. F g. F h. T
i. T j. T k. F l. T m. F n. T o. T p. T

Nothing can stop the man with the right mental attitude from achieving his goal; nothing on earth can help the man with the wrong mental attitude.

Thomas Jefferson